









LO 5 Mechanics

Concepts

- Launch angle.
- Projectile Motion.
- Range.



Skills

- Students should be able to explain the independence of the vertical and horizontal motion of projectiles.
- Demonstrate both mathematically and experimentally that projectiles move in a parabola
- Solve 2-dimensional projectile motion problems

Projectile motion consist of two types of motion 1. Motion in x-axis

2. Motion in y-axis



X-axis is motion is at constant speed And equal $v \cos \theta$

Y-axis motion is not at constant speed due to gravity acceleration and equal v sin θ



All projectile motion laws comes from kinematics equation

 $v = v_0 + at$ no Δx $\Delta x = v_0 t + \frac{1}{2} a t^2$ no v $v^2 = v_0^2 + 2a \Delta x$ no t $\Delta x = \overline{v}t = \frac{1}{2} (v + v_0) t$ no a $\Delta x = vt - \frac{1}{2} a t^2$ no v_0



Case one



Case two



Case 3

T from A to
$$B = \frac{v \sin \theta}{g}$$

 $h = \frac{v^2 \sin^2 \theta}{2g}$
Y max=0.5 g($\frac{v \sin \theta}{g}$)^2
T from B to $C = \sqrt{\frac{2Y \max}{g}}$
 $R = \frac{v^2 \sin 2\theta}{g}$

Trajectory equation
Y = Tan
$$\theta * x - \frac{gx^2(1 \tan^2 \theta)}{vix}$$



Questions

Q1. A ball is projected with velocity 10 m/sec at angle of 30° with the horizontal surface. The speed of the ball after 1 second will be(Use g=10m/sec?)

- a. 5 m/sec
- b. 20 m/sec
- c. 8 m/sec
- d. 10 m/sec
- e. 12 m/sec
- Answer d

Q2. A ball is projected with velocity 10 m/sec at angle 30 with the horizontal surface. The time taken by ball to reach the ground is

- a. 1sec
- b. 2 sec
- C. 3 sec
- d. 4 sec
- e. 5sec
- Answer b

Q3. A ball is projected with velocity10m/sec at angle of 30° with the horizontal surface. The range of the projectile

- is
- a 10m
- b. 10√3m
- c. 20 √ 3m
- d. 30 √ 3m
- e. 40 $\sqrt{3}$ m
- Answer c



Q4. A ball with velocity 10 m/sec at angle of 30° with the horizontal surface. The maximum height attained by the projectile is

- a.5m
- b. 10m
- c. 15m
- d. 20m
- e. 25 m
- Answer a

Q5. A ball is projected with velocity 10 m/sec at angle of 30° with the horizontal surface. The angle made by the line joining point of projection with the point of maximum height is

- a. Tan^-1 $2/\sqrt{3}$
- b. Tan^-1 $1/2\sqrt{3}$
- c. Tan^-1 $\sqrt{3}$
- d. Tan^-1 $1/\sqrt{3}$
- e. Tan^-1 $\sqrt{3/2}$
- Answer b

Q6. The angle of projection for the range of projectile to be equal to its maximum height is

- a. $\theta = \tan^{-1}(2)$
- b. $\theta = \tan^{-1}(3)$
- c. $\theta = \tan^{-1}(4)$
- d. $\theta = \tan^{-1}(2/v3)$
- e. $\theta = \tan^{-1} (1/v3)$
- Answer c



Q7.For a projectile fired with a certain velocity, the Maximum possible Range and the Maximum height attainable are related as

- a. Rumax=2Hmax
- b. Rmax=Humax/2
- c. Rmax=3Hmax
- d.Rmax=Hmax
- e. Rmax=-Hmax
- Answer a
- Q8. A ball projected at an angle θ , attains a maximum height H1 and if the ball is projected at angle (90- θ) and the maximum height attained by the balls is H2 then the range of projectile will be
- a. $R = \sqrt{H1H2}$
- b. R= $\sqrt{H1H2/2}$
- c. R= $2\sqrt{H1H2}$
- d. R= $3\sqrt{H1H2}$
- e. R= $\sqrt{H1H2/3}$
- Answer d
- Q9. the motion of projectile is described by equation $y=ax-bx^2$. the range of the projectile is
- a. a^2/b^2
- b. a/2b
- c. 2a/b
- d. a/b
- e. a/b^2
- Answer d





LO 6 Mechanics



concepts

- Resultant of forces
- resolution of a force
- Lami's rule
- Triangle of force
- Equilibrium of co-planar forces act at a point



skills

Find the resultant of forces meet at a point resolve a force into two component Use Lami's rule use triangle of force Solve equilibrium problems requiring point forces to balance The resultant force between two perpendicular forces



 $R^{2} = F1^{2}+F2^{2}$ F1=R cos θ F2=R sin θ θ =Tan^-1 $\frac{F2}{F1}$ Resultant between two forces



 $\frac{F1}{\sin\theta 2} = \frac{R}{\sin\theta 2 + \theta 1} = \frac{F2}{\sin\theta 1}$ $R^{2} = F1^{2} + F2^{2} - 2^{*}F1^{*}F2^{*}Cos(180 - \theta 2 + \theta 1))$ $R^{2} = F1^{2} + F2^{2} - 2^{*}F1^{*}F2^{*}Cos(\theta 2 + \theta 1))$ $R = \sqrt{F1^{2} + F2^{2} - 2^{*}F1^{*}F2^{*}Cos(\theta 2 + \theta 1))}$ $R = \sqrt{F1^{2} + F2^{2} - 2^{*}F1^{*}F2^{*}Cos(\theta 2 + \theta 1))}$ $Tan\theta 2 = \frac{F2 \sin^{2}\theta 2 + \theta 1}{F1 + F2 \cos^{2}\theta 2 + \theta 1}$









 $\mathbf{Y}_{=R \sin \theta 1 + R \sin \theta 2 + R \sin \theta 3 + R \sin \theta 4}$



Questions

(1) Parallelogram Law of Forces states, "if two forces acting simultaneously on a particle be represented in magnitude and direction by two adjacent sides of a parallelogram, their resultant may be represented in magnitude and direction by

a. its longer side

b. its shorter side

c. the diagonal of the parallelogram which does not pass through the point of intersection of the forces

d. the diagonal of the parallelogram which passes through the point of intersection of the forces

Answer d

(2) The resultant of two forces 'P' and 'Q' acting at an angle '0', is

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a. P 2 + Q2 + 2P \sin \theta
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b. P 2 + Q2 + 2Pcos θ
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c. P 2 + Q2 + 2P \tan \theta
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d. P2 + Q2 + 2Pcos θ
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Answer d

(3) The magnitude of two forces acting on a particle are 5, 8 newton, then the smallest value of their resolution = Newton

a. 2 b.3

c.7

d. 13

Answer b



(4) Following are the angles between two forces when the resultant is maximum and minimum, is ...

a. 90° and 0°

b. 0° and 90°

c. 180° and 0°

d. 0° and 180°

Answer d

(5) A force P of 50 N and another force Q of unknown magnitude act at 90° to each other. They are balanced by a force of 130 N. The magnitude of Q is ...

a. 60 N

b.80 N

c.100 N

d. 120 N

answer d

(6) The magnitude of two forces are 2F, 5F newton and the measure of their included angle is θ and their resultant is 3F, then θ =

a. zero

b. **60**°

с. **90**°

d. 180°

Answer d



(7) If the resultant of forces acting on a body is zero, the body

- a. is in equilibrium
- b. is not in equilibrium
- c. is moving with non-uniform velocity
- d. none of the above
- Answer a
- (8) A point subjected to a number of forces will be in equilibrium, if
- a. sum of resolved parts in any two directions at right angles, are both zero
- b. algebraic sum of the force
- c. two resolved parts in any two directions at right angles are equal
- d. algebraic sum of the moments of the forces about the point is zero Answer a
- (9) Two forces of magnitudes 4 and 6 newton. The measure of the angle between them is 90°, then the tangent of the angle between the resultant and the first force equal
- a. **2 3**
- b. **3 2**
- c. **2 13**
- d. 62
- Answer b



(10) Two forces meet at a point, their magnitude are F1, F2 where $1 \le F1 \le 9$, $3 \le F2 \le 7$ and the magnitude of their resultant R, then

- a. $2 \le R \le 16$
- b. $4 \le R \le 16$
- c. 6 ≤ *R* ≤ 16
- d. $0 \le R \le 16$
- Answer a



LO 7 mechanics



CONCEPTS:-1)Net Force 2)Mass 3)Acceleration 4)Free Body Diagram 5)Friction

Net Force

• The definition:-

The net force is defined as the sum of all the forces acting on an object.

Net force can accelerate a mass. Some other force acts on a body either at rest or in motion.

The net force is a term used in a system when there is a significant number of forces.



• When the body is at rest: $F_{net} = Fa + Fg.$ Where, Fa = applied force, Fg = gravitational force • Net force when a body is in motion: $F_{net} = Fa + Fg + Ff + FN.$ Where, Fa is applied force, Fg is the gravitational force, Ff is the frictional force, FN is a normal force



Mass

The definition
 Mass is the quantity of matter in a physical body.
 An object's mass also determines the strength of its gravitational attraction
 to other bodies.

• The SI unit

The SI base unit of mass is the kilogram (kg). An object on the moon would weigh less than on Earth because of the lower gravity, but it would still have the same mass.

This is because weight is a force, while mass is the property that

(along with gravity) determines the strength of this force.



Acceleration

The definition

• Acceleration is a vector in the same direction as the change in velocity, Δv .

Since velocity is a vector, it can change either in magnitude or in direction.

Acceleration is therefore a change in either speed or direction or both. Acceleration is the velocity in m/s divided by time in s.

• The SI unit

The SI unit for acceleration is m/s2



Free-body diagrams

• The definition

Are diagrams used to show the relative magnitude and direction of all forces Acting upon an object in a given situation The direction of the arrow shows the The direction that the force is acting.





Friction

• The definition

Friction is a force between two surfaces that are sliding, or

trying to slide, across each other.

For example, when you try to push a book along the floor,

friction makes this difficult.

Friction always works in the direction opposite to the direction in which the object is moving, or trying to move.

Friction always slows a moving object down.

The amount of friction depends on the materials from which the two surfaces are made.

The rougher the surface, the more friction is produced.







SOME QUESTIONS

A man is tied to a parachute, descends with his parachute vertically downwards. If the resistance of the air is proportional directly to the square of the magnitude of its velocity, and the resistance of the air equals $\frac{4}{6}$ the weight of the man and the parachute when the velocity = 12 km/h. Find the velocity of descending of the man and his parachute when he moves with uniform velocity.



A car of weight (W) ton.wt. , it can descends with uniform velocity on an inclined plane inclines to the horizontal at an angle of measure θ without using the force of its engine. If the engine of the car is used with its maximum force then it can ascend. The same inclined plane with uniform velocity. Find the force of the engine of the car given that W = 4 ton.wt. , θ = 30° and the resistance does not change in the two cases.

R = 2000 kg.wt , F = 4000 kg .wt

(4) A body is moving with uniform velocity under the action of three forces : $\vec{F_1} = \hat{i} - 2\hat{j}$ $, \overrightarrow{F_2} = (a+1)\hat{i} + \hat{j}$ and $\overrightarrow{F_3} = 4\hat{i} - (b-2)\hat{j}$, then $a+b = \dots$ A (a) - 6(b) - 5(c) 1(d) 6 (5) (2nd session 2017) A body moves in a straight line with a uniform velocity under the action of the two forces $\overrightarrow{F_1} = 2 a \hat{i} - 3 \hat{j} + 4 \hat{k}$ and $\overrightarrow{F_2} = 6 \hat{i} + b \hat{j} - e \hat{k}$, then a + b + e equals AA (d) - 4(c) - 3(a) 4 (b) 3**6**) A horse pulls a wooden mass on horizontal ground with force of magnitude 100 kg.wt. and inclined at 30° to the horizontal. If this mass moves with a uniform Β (b) $50\sqrt{3}$ (c) 100 (d) 100√3 (a) 50

(7) 🛄 If a body of mass 20 kg.wt. slides with a uniform velocity on an inclined plane to the horizontal with angle of measure 30°, then the resistance of the plane in kg.wt. equals В (c) $10\sqrt{3}$ (b) 10 (a) zero (d) 20 (8) \square A body moves with a uniform velocity under the action of three forces F_1 , F_2 , F_3 where $\overrightarrow{F_1} = 5 \overrightarrow{i} + 7 \overrightarrow{j} + 35 \overrightarrow{k}$, $\overrightarrow{F_2} = 5 \overrightarrow{j} + 49 \overrightarrow{k}$, then the magnitude of $\overrightarrow{F_3}$ is force units. С (a) 49 (b) 54 (c) 85 (d) 103

- (1) A car of mass 4 tons moves on a horizontal road with a uniform velocity. If the force of the engine is 120 kg.wt., then the resistance of motion per ton of the mass is
- (b) 30 kg.wt. (c) 120 kg.wt. (d) 480 kg.wt. (a) 4 sec tons (2) A body moves in a straight line under the action of two forces : $\vec{F_1} = -4\hat{i} + 5\sqrt{3}\hat{j}$ $, \vec{F_2} = 7\hat{i} - 4\sqrt{3}\hat{j}$, then the added force that if acts at the body, it will move in uniform velocity is (a) $-3\hat{i} - \sqrt{3}\hat{j}$ (b) $3\hat{i} - \sqrt{3}\hat{j}$ (c) $3\hat{i} + \sqrt{3}\hat{j}$ (d) $-3\hat{i} + 9\sqrt{3}\hat{j}$ (3) A body moves with a uniform velocity in a straight line under the action of the forces $\overrightarrow{F_1} = 2 a \hat{i} - 3 \hat{j}$, $\overrightarrow{F_2} = 6 \hat{i} + b \hat{j}$, $\overrightarrow{F_3} = a \hat{i} + 5 \hat{j}$, then $a + b = \dots$ C

D

(a) zero (b) - 2 (c) - 4 (d) 4



(9) A car of weight 5 tons wt. descends (without the engine) along a plane inclined to the horizontal at an angle of $\sin = \frac{1}{50}$ with uniform velocity and if the engine works then it ascends this plane with uniform velocity, then the force of the engine of the car = …… kg.wt. D (a) 50 (b) 100 (c) 200 (d) 400 (10) A body of weight 9 newton is placed on a horizontal plane, it moved with uniform velocity under the action of two forces $\overrightarrow{F_1} = 3 \hat{i} + 4 \hat{j}$, $\overrightarrow{F_2} = -\hat{i} + \hat{j}$ where \hat{i} is a unit vector on the plane in the direction of motion and \hat{j} is a unit vector perpendicular to the plane, then the normal reaction = newton. (a) 2 (b) 4 (c) 5 (d) 9D

Scalar VS Vector



Scalar quantity: it is a quantity that can be determined by the magnitude only. And doesn't need the direction to determined.

EX: mass, time, speed, and distance between two points.

Vector quantity: it is the quantity that must be determined by the magnitude and the direction.

EX: force, velocity, and displacement.

Vectors Summation

The resultant of forces is considered as a summation of vectors. The rule: \Box

$$\mathsf{R}=\sqrt{f1^2 + f2^2 + f1f2\cos\alpha}$$

Special cases:

1)If the two forces are perpendicular ($\propto = 90^{\circ}$)

$$R = \sqrt{F_1^2 + F_2^2}$$



- 2) If the two forces are equal in magnitude (F1 = F2 = F):
- $R=2F\cos \propto /2$
- 3)If the two forces have the same line of action and the same direction ($\propto = 0^{\circ}$):

 $R = F\mathbf{1} + F\mathbf{2}$

- 4) If the two forces have the same line of action but in opposite directions ($\propto = 180^\circ$): R = |F1 - F2|
- 5)If the two forces are equal in magnitude and have the same line of action but in opposite directions:

R = zero



The direction of force and acceleration vectors can be considered by using the x and y components. ay is the acceleration on the Yaxis and ax is the acceleration <u>on the X-axis</u>. $a_x = \sum F_x \quad a_y = \sum F_y \quad \overrightarrow{a} = \sqrt{a_x^2 + a_y^2}$ $\overrightarrow{m} \quad \overrightarrow{m}$

So, if you get X and Y component you can know the velocity and distance

 $\sum_{i=1}^{n \text{ of } f_{i}} \vec{F} = m\vec{a}$

is equivalent to



In the motion of the body, the acceleration is varying A- Directly as the force and inversely as the mass B- Directly as the force and directly as the mass C- Directly as the force D- Inversely as the mass Answer: A

A body of mass 130 gm is placed on a smooth horizontal The table is connected by a string passing over a small smooth pulley at the edge of the table. A body of m gm is suspended at the end of the string if the system is left to move to start from rest with uniform acceleration 3.43m/sec2 ,then m =

A- 0.7 kg

B- 140 gm

C-70 gm

D- 34 gm

Answer: C

Two bodies weighing 45 N and 60 N are huge to the ends of a rape passing over a frictionless pulley, the tension in the string equals

A- 42.51 N

B- 51.42 N

C- 45.36 N

D- 49 N

Answer: B



A block of mass M is kept on a smooth inclined plane of inclination θ . The force exerted by the plane on the block has a magnitude:

(a)*mg*

- $\cos \theta$
- (b) *mg*
- (c) $mg \tan \theta$
- (d) $mg \cos\theta$

Answer: D

The force exerted by the lift on the foot of a person is more than his weight, the lift is:

(a) going up and slowing down

(b) going up and speeding up

(c) going down and speeding up

(d) going up with uniform velocity

Answer: B



Laws of circular motion

At the first, you know that the movement of bodies does not have to remain a line, and any body can walk in a straight line, or it can walk in a curve and if the curve turn, then it is called circular motion.

As:

- motion of planets around sun.
- motion of moon around a planet.
- motion of electrons around nucleus.

So, where did circular motion come from?

It comes from the effect of force on the object that moves, but the effect of this force remains strange (a little different).



through study of newton second law, you have learned that:

When a net force acts on a moving body, its velocity changes which means that it acquires acceleration, and the change in velocity depends on the direction of the acting net force relative to the direction of motion.

If the direction of net force is:

(1) In the same direction of motion.

» the speed of the moving object increases, and the direction of motion does not change.

(2) In opposite direction of motion.

» the speed of the moving object decreases, and the direction of the motion does not change.

(3) Perpendicular to the direction of motion.

» the speed of moving object remains unchanged, but the direction of motion changes (centripetal force).



So, what is the difference between uniform circular motion and centripetal force?

uniform circular motion:

it's the motion of a body in a circular path with a constant speed and a changeable direction.

centripetal force:

it's the force acting continuously in a direction normal to the motion of the body, causing it to move in circular path.

And what is the centripetal acceleration? Its acceleration acquired by an object while moving in a circular path due to the change in direction of its velocity.



Centripetal acceleration

Periodic time: it's the time taken by a body to make one complete revolution.

T = t / n $V = 2\pi r / T$ Angular velocity = $2\pi / T$





Deducing the centripetal acceleration (*ac*)

If the body moves in a circular path from point A to point B as in the figure, the direction of its velocity (V) between the two points Changes while the magnitude/of its velocity remains constant, thus, the change of velocity (V) is resulted due to the change in direction only. Therefore, ac = V_2 / r = $4\pi_2 r / T_2$

V B V r t 0 V C r

Tangential velocity :

centripetal acceleration is directly proportional to the square of the tangential velocity.

Slope = \triangle ac $/\triangle V_2$





The radius of the circular path:

Centripetal acceleration is inversely proportional to the radius of the circular path. Slope= $\triangle \mathbf{ac} / \triangle (\mathbf{1/r})$ $\mathbf{a_c}$

Centripetal force

When a centripetal force Fc acts on a body of mass m to make it rotate in a circular path with a centripetal acceleration ac , so according to newtons second law the force is given by the

Relation: $\mathbf{F} = \mathbf{ma}$ Therefor, $\mathbf{ac} = \mathbf{V}_2 / \mathbf{r}$

 $Fc = m ac = mV_2 / rThe object mass:$

The object mass:

Centripetal force is directly proportional to the objects mass. $Slope = \triangle Fc / \triangle m = V_2 / r$ F_c



The tangential velocity:

Centripetal force is directly proportional to the square of the tangential velocity.

Slope = $\triangle \mathbf{Fc} / \triangle \mathbf{V_2}$



 \mathbf{F}_{c}



Radius of circular path:

Centripetal force is inversely proportional to the radius of circular path. $Slope = \triangle Fc / \triangle (1/r)$

Free body diagram

If we have a system of forces and we want to analysis this force react on which direction and how much magnitude so, scientists came with the free body diagram.

Free body diagram: it represent an analysis of forces in a system of forces.

 $\sum \mathbf{F} = \sum \mathbf{F} \mathbf{x} + \sum \mathbf{F} \mathbf{y}$



Questions:

1 A ball is attached to a string that is 1.5m long. It is spun so that it completes two full rotations every second. What is the centripetal acceleration felt by the ball?

2 An amusement ride is used to teach students about centripetal force. The ride is a circular wall that you place your back on. The wall and floor then begin to spin. Once it reaches a certain rotational velocity, the floor drops, and the students are pinned to the wall as a result of centripetal force. A student of mass 50kg decides to go on the ride. The coefficient of static friction between the student and wall is 0.8. If the diameter of the ride is 10m, what is the maximum period of the ride's

rotation that will keep the student pinned to the wall once the floor drops?

3 Candy companies have long strived to catch the attention of children. One item that does this particularly well is the gumball machine. A certain gumball machine has a column that is 1.5m tall with a spiral track of radius 0.25m on which the gumball travels. The slope of the track is 10° and the average frictional force exerted on the gumball as it travels down the track is 0.05N. What is the centripetal force on a gumball of mass 0.1kg as it reaches the end of the track?

4 A 25kg boy is riding a merry-go-round with a radius of 5m. What is the centripetal force on the boy if his velocity is 6m/s?

5 A ball of mass 1kg is on a string of length 2m. If the ball is being spun in vertical circles at a constant velocity and with a period of 2s, what is the maximum tension in the string?



5 A car of mass m is driving around a circular track of radius r at a constant velocity of v. The centripetal force acting on the car is Fc. If the car's velocity is doubled, what is the new centripetal force required for the car to drive on the circular track?

7 Imagine a car driving over a hill at a constant speed. Once the car has reached the apex of the hill, what is the direction of the acceleration?

8 Imagine a table that has a hole drilled through the center. Through this hole is a string that connects one mass that hangs under the table, to a mass that is on top of the table and spinning. mass M weight 10 kg, mass m weighs 5 kg and the radius, R, of the circle mass M traces out is 50 cm. At what speed, v, does the mass M need to rotate at in order to keep mass m suspended?

9 A 10kg object moves in a circular motion with a diameter of 5.1meters. What is the magnitude of this object's velocity?

10- At the 2015 NCAA Track Championships, a competitor in the hammer is in the midst of his throw. During a period of his pre-throw spinning, his hammer is moving at a constant angular velocity of

25.00m/s in its circular orbit. Being a regulation hammer, it weighs 7.257kg, and the length of its rope is

121.5cm. What is the magnitude of the force exerted towards the competitor along the hammer's rope?

Answers

237 m/s² 2) 3.97 s 3) 8.5 N 4) 180 N 5) 29.7 N 6) 4Fc
 Downwards 8) 1.6 m/s 9) 5 m/s 10)3733 N

